In *Contemporary Psychology* (1961), **6** (2): 75–76.

The Psychology of Set Theory

Léo Apostel, A. R. Jonckheere, and Benjamin Matalon

Logique, apprentissage et probabilité. (Etudes d'Epistémologie Génétique, VIII.) Paris (108, Boulevard Saint-Germain): Presses Universitaires de France, 1959. Pp. 186. 900 fr.

Reviewed by PATRICK SUPPES

As contributors to the Études d'Épistémologie Génétique *the authors place themselves in the important and influential group stimulated by the thought of Jean Piaget in Geneva. Apostel is a professor of philosophy in Belgium. Jonckheere lectures on psychology in University College, London. Apostel and Matalon contributed to the third and fourth* Études (CP, *Oct. 1958, 3, 310–312*). *Dr. Suppes, the reviewer, is a logician and philosopher of science at Stanford University. He has published* Introduction to Logic (*1957*), Axiomatic Set Theory (*1960*), *with Donald Davidson,* Decision Making (*1957*), *and with Richard C. Atkinson,* Markov Learning Models for Multiperson Interactions (*1960*). *He is pretty well psychologized from being a Fellow at the Center for Advanced Study in the Behavioral Sciences (1955–1956), from association with W. K. Estes, and from his interest in learning theory, theory of measurement, and decision making.*

THIS book is Volume VIII in the series *Études d'Épistémologie Génétique,* published under the direction of Jean Piaget, and many of the themes in the present work are familiar from earlier volumes in the series. As is the custom in this series, the volume is not written jointly by the three authors, but actually consists of three separate articles, by far the longest of which is the first one by Apostel (138 pp.). It is entitled *Logic and Learning,* and from casual references in later volumes I would judge Piaget thinks highly of it as a theoretical contribution.

The task Apostel sets himself is an exciting and interesting one. He wants to give a behavioristic account of the logical notions of class, relation, proposition, and inference, as well as of the logical constants like *and* or *or*. He begins (Part I) by attempting to provide psychological definitions of these five notions. For example, a class exists for a subject if, roughly speaking, he makes the same response when presented with any member of this class, but does not make the response to any object not in the class. This is not exactly what the author says, but one of my difficulties is that I cannot understand precisely how the author does want to define the

psychological existence of classes (or relations or inferences). After a promising beginning, Part I is disappointing. I doubt that it would be clear to anyone how to design an experiment to test the intuitive adequacy of the definitions proposed.

In Part II Apostel gives an admittedly routine survey of the 'laws' of learning. What he does is to sketch the theories of Guthrie, Tolman, Piaget, and Hull, in that order. His intention is to provide background for the theoretical analysis in Part III of the definitions introduced in Part I. What is surprising is that he tries to bring together all four of these theories in terms of the linear response models of Bush and Mosteller. Unfortunately the attempts to extend these models to the theories in question are too qualitative and vague to be regarded as serious contributions to the development of the theories or the literature of mathematical models.

For the same kind of reasons Part III also fails in its mission, but some of the discussion centering around the application of Hull's theory to classificatory behavior is interesting and worth reading. The same thing cannot be said for the section which gives a psychological interpretation of axiomatic set theory. I would characterize this discussion as fanciful. To give two examples, the author claims that the technical distinction between sets and proper classes in von Neumann set theory (which does not exist in Zermelo set theory) is reflected in the classes defined psychologically. Secondly, the psychological interpretations given the axiom of infinity and the axiom of choice are in reality scarcely connected with their literal mathematical meaning. The axiom of infinity is said to be 'verified' to the extent that new classificatory divisions can be constructed without limitation. This is perhaps close to Aristotle's idea of the potentially infinite, but this kind of infinity is already present in set theory without the axiom of infinity, as was pointed out by Zermelo in 1909 when he showed that elementary number theory can be constructed within set theory without this axiom.

The important point highlighted by the analysis of set theory is that Apostel tries to give a psychological interpretation of logic and the theory of sets without any intervening general analysis of language and meaning. This would seem to be clearly a mistake. Logic and set theory are the end products of a long and sophisticated verbal development. It seems unlikely that a seriously detailed behavioristic interpretation of them can be given in terms of nonverbal responses. The connection of set theory with the empirical world, let alone the behavior of organisms, is at best highly tenuous and indirect.

THE two shorter articles, one by Jonckheere and the other by Matalon, discuss various aspects of stochastic models for learning. Jonckheere gives a succinct review of the usefulness of stochastic models in dealing with experiments in which one of two responses is possible. He then goes on to propose an urn model which generalizes the usual linear response models. Actually I have myself never really seen the point of urn models in the present context. They provide one simple realization of discrete probability processes, but it is not a realization which is highly suggestive in terms of psychological theory. Jonckheere is unduly pessimistic in his discussion of the problems of parameter estimation and goodness-of-fit tests for stochastic models. When linear response models are replaced by stimulus sampling models, simple maximum-likelihood estimation procedures are often available. And the difficulty of non-independence of observations in applying goodness-of-fit tests disappears if transitions in responses rather than responses themselves count as the observations.

Matalon analyzes sympathetically certain properties of current stochastic models which give rise to problems—for example, their very restricted capacity for incorporating memory. He also makes some suggestions for extension but does not work out the details.

It should perhaps be remarked that none of the three articles seriously considers relevant experimental data.