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The Concept of Set

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HISTORICALLY, the theory of sets is a relatively recent arrival on the scene in mathematics. But its chronological position in the development of the branches of mathematics does not imply that the concepts with which it deals are for the expert mathematician only.

The concept of set is neither too abstract nor too complex for the primary-grade child. It is, in fact, quite easy to explain and to comprehend. The term "set" in mathematics simply refers to any family, group or collection of objects or entities.

The teacher may question the place of the set concept in the school mathematics curriculum. Does it belong at all and, if so, where?

There are sound and valid reasons for the increasing interest in and experimentation with the introduction of sets at the elementary level. These reasons have their roots in significant developments at the very foundations of modern mathematics and in important relations of the theory of sets to all branches of mathematics.

The study of sets constitutes an excellent introduction to mathematics for the child The elementary notions from set theory can and should be an integral part of the sequence of the development of concepts of number and number operations, such as addition, subtraction and multiplication.

The argument for developing the set concept at the beginning of the primary grades is two-pronged. Its basis is logical on the one hand and psychological on the other. Logically, sets precede numbers. Furthermore, all mathematics can be developed from the notion of sets because with few exceptions the entities studied in mathematics may be regarded as particular sets or classes of objects.

The prior introduction of sets permits other mathematical concepts such as number, and operations on number, to be presented in an unambiguous way. It permits clear and precise definitions where too often we have been forced to make vague and imprecise partial explanations. For example, does the primary-grade child's understanding of arithmetic allow him to give an unambiguous answer to the question: What is a number? Do the concepts and notation traditionally introduced to first-grade children permit us to span the vast gap in abstraction between physical objects and Arabic numerals in any precise and meaningful way? The logical relation of set to number makes it possible to answer these questions simply and with precision.

Psychologically, the study of sets is an ideal place to begin with children. Sets are concrete, numbers are abstract. The notion of a collection of objects is certainly a familiar one to any child. At the same time, the combining or separating of sets is more meaningful to the child than dealing with abstract numbers. The putting together of sets of physical objects, for example, is concrete; the addition of numbers is abstract. Considerable empirical evidence is being compiled indicating that concepts of set, set operations and set relations are readily and easily comprehended by first-grade children.

For years, primary-grade teachers have recognized the necessity for concreteness in teaching mathematics. They use objects displayed in groups as a preparation for the learning of number concepts. A group of objects constitutes a set. Thus the method of displaying problems by grouping objects approaches the set notion, but the lack of an explicit notation makes precise definitions difficult, if not impossible. As valuable as the initial work with objects is, the critical step in the child's mathematical understanding is the move from the concrete to the abstract realm of number and number description.

The introduction of an explicit notation for sets

Turn to page 86

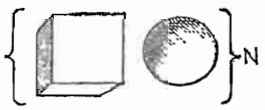
Continued from page 51

makes it possible to describe the transition levels of abstraction between object and number name or numeral. The relationship between a set or a group of things and the name of a number associated with that set can be made clear in a way not permitted by verbalization at the primary-grade level.

The introduction of the set concept makes it possible to teach a precise and mathematically exact definition of number which is meaningful to the child. Children who have learned the notion of property through discussion of familiar properties things have, such as color, shape, texture, etc., learn readily that number is a property of a set. Continual emphasis on this definition precludes children's thinking of numbers as meaningless symbols or addition as an operation performed on meaningless number symbols. Numbers are properties of sets, and the operation of addition of numbers is a general way of combining sets of things without paying any real attention to the things themselves.

The levels of abstraction encountered in moving from objects to number may be made explicit in the following way. We begin with a set of physical objects, perhaps a ball and a block. The ball and the block are the members of the set. *The first level of abstraction is the description of the set* This is accomplished by picturing the set members and enclosing the pictures in braces ("curly braces"). This notation is conventional in set theory:

The next level abstracts from the particular character of the set members to the single property of number. For notation the letter "N" is put before the same set description:



This notation names the number two just as the numeral "2" does, but at the same time it maintains the pictorial character of the set description. This is particularly important because it makes explicit the precise relation between a set consisting of a ball and block and the number associated with that set. It represents to the child *a more abstract level because here he is interested not in what the set members are but only in how many there are.*

The final step is the Arabic numeral which also names the number. The

Turn to page 88

Continued from page 86

child learns that either notation names the number which is the property of the given set, and thus the tendency to focus on the numeral as being number itself is avoided

Through the use of the notation just described, the correspondence between the easily comprehended concrete operations on sets and the more abstract operations on number can be introduced at the beginning of the child's mathematical experience The meaning of abstract operations such as addition and subtraction of numbers is made more clear by presenting operations on sets as concrete analogues to numerical operations.

The operation of union of sets, putting sets together to form a new set, is analogous to the addition of numbers. The union of sets is first described, then addition of numbers as described by the N notation, and finally the addition of numbers using Arabic numerals. In this manner, the child considers the addition of numbers without commitment to the particular notation of Arabic numerals Stress is on the meaning of the operation and avoidance of rote memorization Moreover, the child can be encouraged to recognize not only the relation between combining groups of things and adding numbers, but also the important distinction between these operations as well. In a like manner, the operation of difference of sets may be presented as analogous to the subtraction of numbers

These ideas and methods are currently being tested by an experimental program carried on in Stanford University's Institute for Mathematical Studies in the Social Sciences In 1959-60, four first-grade classes, and in 1960-61, twenty-five first-grade classes began a program using the approach discussed above A series of workbooks, *Sets and Numbers*, was developed and these materials constituted the arithmetic program for the experimental classes. The groups were taught by the regular classroom teacher; no special background was required.

This year the program has been expanded with one hundred first- and second-grade classes participating. Results from the first two years' experimentation show that notions of set and set operations are more easily comprehended than those of number A special point of some interest is the striking evidence during this experiment for the early introduction of zero as the number of things in the empty set. An achievement test covering the number concepts common to all primary workbooks has provided evidence that the experimental classes were signifi-

Turn to page 90

Continued from page 88

cantly superior in knowledge of number concepts and operations to comparable classes using more conventional approaches to arithmetic. Continuation of the program into other grades will make it possible to evaluate the achievement of the goal of deeper understanding of mathematics and its structure.