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SET THEORY IN THE PRIMARY GRADES

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Those very critical years in learning any subject, the first three or four years of school, are now coming in for their share of the considerable attention being paid to curriculum revision in mathematics. Often ignored in revision and upgrading of mathematics teaching and materials, the primary grades are being recognized as the stage at which a strong foundation in mathematics can be and should be laid. But at this point, experimentation in new mathematics programs in the primary grades has not been as extensive as new programs at other levels of the elementary and secondary school.

One of the experimental projects that began in the first grade and has thus far concentrated attention on the primary grades is The Sets and Numbers Project in Elementary School Mathematics at Stanford University. It is completing three years of classroom experimentation and this article presents a summary of its objectives and its progress.

Objectives and Content

In general, the major objective of the project is to develop and test a new mathematics curriculum for kindergarten through grade six. At the present time materials have been prepared for kindergarten and primary grades. Another grade is added each year in the program of classroom experimentation. The intention is to provide a program that is both mathematically sound and pedagogically simple, a program that stresses structure and foundations.

Although the major emphasis is on the development of the concepts, laws, and skills of arithmetic, considerable content from other branches of mathematics is added. For example, the inclusion of a substantial body of content from geometry is introduced by work with simple geometric constructions in the first, second, and third grades. As another example, by introducing the use of letters as variables, the program intends to include the basis for a smooth transition to the study of algebra. In presenting letters as variables in a simple context that requires no technique of solution, a familiarity with algebraic variables may be developed at the earliest stages of the child's mathematical education.

While the project takes the point-of-view that there are many sound and valid arguments for the addition of more and different content in elementary school mathematics, and that evidence is ample that children of this age *can* learn much more

mathematics than traditionally assumed, a goal of greater importance is that of providing a stronger foundation in arithmetic. It is believer that arithmetic can be taught with an emphasis on concepts, on structure and logical development, on laws, without sacrificing the development of skills. Thus the goal is to deepen as well as to extend mathematical experience of the child.

Another objective is to encourage precise and exact mathematical language. Vague and ambiguous terms are avoided and technical vocabulary is used where appropriate. Experience has shown that technical vocabulary is easily learned by the young child when the idea represented is clear and explicit.

Particular attention is paid in the material to the sequence of development of concepts. The attempt is to move from the concrete to the abstract, from familiar ideas to now ideas in a series of small steps, each one building on the previous one.

The Concept of a Set and its Role

The central concept in the project materials developed for primary grades is that of a set. It is the foundational and unifying idea throughout. This concept is basic to the development of the idea of a number and operations on sets are introduced as fundamental to the parallel operations on number.

There are at least two major reasons to begin first-grade arithmetic with the explicit introduction of the notion of set and appropriate notation for sets and operations upon them. In the first place, sets are more concrete objects than numbers. At the same time operations upon sets are more meaningful to the child than manipulation of numbers. The putting together of sets of physical objects, for example, is a more concrete operation than the addition of numbers. The many exercises in the grouping of objects displayed in current books is in fact a recognition of the greater concreteness.

In the second place, the prior introduction of sets and additional explicit notation permits mathematically exact and precise definitions and concepts rather than the often vague and ambiguous notions encountered in explanations of relations between concrete groups of objects and the Arabic numerals. For example, students can learn a clear, simple, and meaningful characterization of numbers as properties of sets. Children who have learned the notion of a property can learn that a precise answer to the question "What is a number?" is, "A number is a property of a set."

The introduction of explicit notation is intended to make the concept of number clear. Ordinarily, we can only assume understanding of the relationship when we make the great leap

in abstraction from groups of objects to numerals which name their particular number properties. The use of set notation allows the steps in abstraction to be made explicit. The first step in abstraction is describing a set in the following way:

The next step in abstraction is the N notation:

This notation names a number but at the same time it maintains the pictorial character of the set description. In this sense it may be considered a "transition" to the Arabic numerals. Here we have abstracted from the particularity of the objects in the set to the single property of number. We must assume that children understand this step if they are to have any understanding of the way a number is related to a set of objects. The notation makes the step clear and precise in a way not permitted by verbalization at the primary grade level. The final step is to the Arabic numeral which in our example is:

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Thus the explicit notation for sets introduces the student at the very beginning of his mathematical experience to the easily comprehended operations on sets—rather than to the more difficult and more abstract operations on numbers. Moreover the introduction of a notation for sets permits consideration of addition and subtraction of numbers without commitment to the particular notation of Arabic numerals.

Throughout the beginning materials, set operations are presented as concrete analogues to numerical operations. Union of sets and addition of numbers are presented in sequence, difference of sets and subtraction of numbers are presented in sequence, subsets and inequalities are presented in sequence, et cetera. The operations and relations of arithmetic are based upon and developed from the foundational concepts of operations on sets and relations between sets. Particular attention is given to mathematical sentences, equations, and the translation of English sentences expressing quantitative relations into mathematical equations. Introduction of commutative, associative and distributive laws is an important part of the third-grade content.

It should be emphasized that it is not our intention to present an isolated body of mathematical content, but rather to present basic concepts and mathematical tools with which an integrated program of learning mathematical tools can be constructed.

Finally, it is believed that the introduction of such basic concepts as those of set and set operations will lead to greater understanding of the structure of the mathematics to be learned.

Background and Present Status

Classroom experimentation in the Sets and Numbers project has covered a period of three years. Prior to this, a pilot study in 1959-1960, involving four first-grade classes, was instrumental in the development of the first experimental materials.

During the academic year 1960-1961, twenty-five first-grade classes in the San Francisco Bay Area were included in the experimental program. Each of the classes used two workbooks, *Sets and Numbers, Book 1A* and *Sets and Numbers, Book 1B* as the total arithmetic program for that year.

The classes were taught by the regular classroom teachers. No special training or background in mathematics was presupposed. The teachers met for a general orientation at the beginning of the school year and approximately monthly thereafter for a discussion of progress and for purposes of introducing new materials to be covered. On the basis of the first year's results and, in particular, the comments and suggestions of participating teachers, the first-grade books were revised for use the following year.

During the academic year 1961-1962, the same classes continued as experimental groups in the second grade and twenty new second grade classes were added. Two books were developed for these classes to test. They were *Sets and Numbers, 2A* and *Sets and Numbers, 2B.*

The program for first grade was considerably expanded. Eighty first-grade classes tested the revised Books 1A and 1B during 1961-1962.

The teacher training program again consisted of general orientation meetings at the beginning of the school year and monthly meetings thereafter. A major portion of each meeting was devoted to discussion of the materials with particular attention to suggestions for improvement. This resulted in revised versions of each of the second-grade books.

During the current academic year, 1962-1963, the program has included first, second, and third grade classes. In addition, a basic book for kindergarten was prepared and is being used experimentally. At the present time, the experimental project

involves 110 first-grade classes, 102 second-grade classes, 68 third-grade classes, and 20 kindergarten classes.

The usual teacher training program of orientation meetings and workshops, and meetings throughout the year has been supplemented by a course in background mathematics for teachers given at Stanford University as a part of the project.

Evaluation

One of the most difficult taks of experimental projects in curriculum revision in mathematics is that of evaluating results. Many of the predicted and hoped-for outcomes are long-term objectives not necessarily immediately apparent. More critical to a program of evaluation, perhaps, is the difficulty in finding appropriate standardized achievement tests. With new content and new objectives, the available tests simply do not provide evidence concerning many of the goals of modern mathematics programs. This problem is greatly increased at the primary-grade level where it is underscored by the usual difficulty in measuring achievement in the absence of completely adequate skills of reading, writing, and comprehending verbal instructions.

Yet any experimental project must evaluate by objective measures. As valuable as the subjective evaluations of experienced teachers are (and these may well be the most important evaluation at this stage), they should be supplemented by objective measurement where possible.

The program of testing in the Sets and Numbers Project has consisted of a variety of types of tests. First, achievement tests are given all experimental classes at the completion of each book in the series. These tests assess the ability of the children to learn content covered in the materials and also provide evidence as to specific difficulties in learning particular concepts and skills.

Detailed results will not be given here but in general the achievement level on these tests is quite high. Toble 1 gives the mean of scores on tests covering Books 1A and 2A. Specific analysis indicated that concepts of set and set operations are less difficult than number and number operations.

Table 1

Achievement Tests on Books 1A and 2A, 1960-61, 1961-62

Year	*Test*	*N*	*Total Possible*	*Mean*
1960-61	1A	620	73	61.40
1961-62	1A	1803	80	68.44
1961-62	2A	893	140	120.90

Although considerably more mathematics was presented to and, as evidenced by the test described, learned by children in experimental classes, the question reasonably is raised "Does the addition of content and emphasis on concepts mean a sacrifice in skills and techniques which have been traditionally the goals of arithmetic teaching?" To determine how well experimental classes can perform on traditional content, standardized achievement tests were administered to a group of randomly selected experimental classes and to matched control classes within the same districts. Classes were matched by administrators within each district on the basis of known variables such, as student ability level, socio-economic level of neighborhood, staff capabilities. Children in control classes had been in a traditional program all year. The test was the arithmetic portion of the Metropolitan Achievement Test, Primary I battery (1st grade) and Primary II battery (2nd grade).

Tables 2 and 3 give results of the two tests and the significance of difference between means.

Table 2

Grade 1	*Exp. Group*	*Control Group*
Number	316.	311.
Mean Score	58.74	54.96
Median Score	60.	56.

Difference of Means Significant Beyond .001 Level
in Favor of Experimental Group
t value = 5.50

Table 3

Grade 2	*Exp. Group*	*Control Group*
Number	287	208.
Mean Score	55.7	56.0
Median Score	55.8	56.0

Difference of Means *Not* Significant
t value = .38

The standardized test results indicate that children using the *Sets and Numbers* materials scored at least as well or better on the traditional content as children who had been in traditional programs. In addition, a considerable body of content not tested by standardized tests was taught.

The relatively better results for the experimental group in the first grade appear to the project staff to be due primarily to two factors. One is the greater experience of the first-grade teachers in presenting project material to their classes. The other is the greater emphasis in the second-grade *Sets and Numbers* books on mathematical content and concepts not examined in the Metropolitan or other standard Achievement Tests. This means that the second-grade children in the project spent a relatively larger amount of time learning material on which they were not tested.

Future Plans

Our future plans call for continued experimental pedagogical experimentation with the concept of set theory in the primary grades. For the coming academic year we are planning to produce augmented editions of the Sets and Numbers books in order to test the efficacy of presenting primary grade children with a very much enlarged body of curriculum materials. (The present books for the first-grade program contain 336 pages. In the augmented edition we are planning on a minimum of 600 pages.) We shall be able to report in the future whether or not considerable augmentation of curriculum materials have a significant effect on learning.

The second idea we would like to attempt to implement during the coming academic year is the idea of having the children make continual active behavioral responses during the usual period the teacher is introducing a new concept. Two kinds of observations and conclusions lead to this suggestion. The one is that with primary children the introduction of new concepts is very heavily dependent upon interaction of an extensive kind with the teacher. The second kind of observation is that in the ordinary discussion activity carried on by the primary teacher it is easy for a small percentage of the students in the class to dominate the discussion with their verbal responses. The aim of this part of our program is to see to what extent the introduction of new concepts in the mathematics curriculum can be facilitated by the use of rather carefully constructed response systems of the following sort. As part of her introduction to new concepts the teacher would use a slide projector. Each child would have on his desk a response panel consisting of three or four buttons and he would be required to select the appropriate answer as shown on the slide. The elicitation of a response would be called for by the teacher with the beginning initial discussions of any new concepts, whether it be identity of sets, union of sets, difference of sets, addition of numbers, multiplication of numbers, etc. The teacher would have beside her on the table with the slide projector a panel that would show the response of each child together with a totalizer indicating how many children had answered the question correctly. Her own progress through the introduction of the concept and the widening of the basis of the discussion would depend on the class as a whole reaching a certain criterion of performance. What is particularly interesting to us is the possibility of the teacher's being able to control her own movement from one stage of the introduction of a concept to another on the basis of the class as a whole showing the criterion mastery of the concept. If our plans for the construction of such

student response systems are successful, we hope to be able to report on their evaluation in the next year.

It should be mentioned that the writing of the workbooks has been supported by the Carnegie Corporation of New York, and the program of classroom evaluation by the National Science Foundation.

Finally, another related program of research is perhaps worth mentioning here. It is a program of psychological research on mathematical concept formation in young children being conducted under the sponsorship of the Office of Education. Much of this research is closely geared to the experimental teaching program in arithmetic. We have been particularly concerned to make a detailed analysis from the standpoint of learning theory of how children of ages 5 and 6 acquire, in a highly structured psychological experiment, the concepts of two sets being identical in the sense of ordered sets, the concept of two sets being identical in the sense of unordered sets, or the concept of two sets being equipollent, that is, having the same number of elements. An overview of some of the psychological experiments conducted during the past three years may be found in an article by Patrick Suppes and Rose Ginsberg in *Science Education,* Vol. 46 (1962), pp. 230-240.