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Individual Choice Behavior: A Theoretical Analysis. *R. Duncan Luce.* New York: John Wiley and Sons, 1959. Pp. xii, 153. $5.95.

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IN THE midst of several Social Science Research Council activities sponsored by the Committee on the Mathematical Training of Social Scientists at Stanford University in the summer of 1957 there circulated a long mimeographed report by Duncan Luce which we called the "red menace" because of the color of its cover and because of the many lively discussions of its contents. That report has been expanded into the present book, and I recommend it to all who are interested in the theory of choice behavior.

The book consists of five chapters and four appendices. Chapter 1 sets forth the basic theory. The heart of this chapter is Axiom 1 introduced on p. 6. The first part of the axiom states that if R is a subset of S, and S is a subset of T, then when a responding organism is faced with the choice of one of the alternatives in the set T, the probability he will select one from the subset R is equal to the probability he will select one from R given a choice from S times the probability he will select one from S given a choice from T. In more perspicuous mathematical language, $P_T(R) = P_S(R)P_T(S)$. The reader will note at once that this equation holds for conditional probabilities, but it is important to emphasize that this part of the basic axiom is not a simple truth about conditional probabilities, for the actual choice set (S or T) changes, not simply knowledge about the subset from which the choice was made. I like to think of the matter this way. Suppose we have an experiment in which subjects respond by pushing one of three keys. In any theory of choice behavior we want to be able to predict the conditional probability of choosing one of the keys, given that the choice was made between two specified keys. This prediction simply deals with one aspect of the three-key experiment. Luce's axiom is concerned with quite another matter. Essentially, his axiom may be used to make predictions about a second experiment, given results on the first. In this second experiment we block one of the three keys and force the response to be one of the remaining two. Some axiom of behavior is required to predict behavior in this new situation.

The second part of Axiom 1 also bears on this experiment. It simply says that the probability of selecting an alternative from any subset S given T remains constant if we delete from T an alternative x which is dominated in preference by some y in T. (In the formal statement of this part of the axiom on p. 6 there is an error in the position of the quantifier binding "x," but the interpretation following makes the intended meaning clear.) As is apparent, the second part of the axiom is particularly apposite to the sequence of two experiments just described.

Axiom 1 comes very close to expressing in a new form a much discussed principle of the last decade, namely, the principle of the independence of irrelevant alternatives. Essential use of this principle was made by Arrow in the proof of his impossibility theorem for social decision functions. Luce's version mainly differs from Arrow's in being probabilistic in formulation, but the more important thing is that he has exploited its consequences in new and surprising directions, which are marked out in the remaining four chapters.

Chapter 2 is concerned with applications to psychophysics. A gloss of the six cases considered is not easily given. They range from Fechner's law to signal detection and rank orderings. The simple yet highly ingenious applications of the basic axioms in this chapter depend on the construction of a scale $v(x)$ for alternatives defined in the following way:

$$P_S(x) = v(x) / \sum_{y \in S} v(y). \tag{1}$$

This v-scale, which is unique up to a similarity transformation, is the main tool used to attack the problems in all the remaining chapters, not just in Chapter 2. It is apparent from (1) precisely how the basic axiom enters, namely, because it is postulated that choice probabilities for different sets of alternatives satisfy the principle of independence of irrelevant alternatives, $v(x)$ may be consistently defined independently of S. Luce's generalization of v to a function of two variables in order to treat interaction of psychophysical continua is particularly elegant, all the more so because of the relative neglect of this topic in the psychophysical literature.

Chapter 3 deals with applications to utility theory. Additional axioms are introduced in this chapter; by and large they are similar in spirit to those of von Neumann and Morgenstern, but there are important differences. Luce is not sympathetic to the expected utility hypothesis, and he feels that an over-emphasis on this hypothesis has had a negative effect on descriptive psychological theories of choice behavior. The difficulty with his alternative approach in terms of his concept of a decomposable preference structure is that the results are too specific not to be falsified by much actual choice behavior. For example, Theorem 12 asserts that if pure alternatives are not perfectly discriminated, then there are exactly three equivalence classes of mixed alternatives, i.e., of alternatives that are probability mixtures of pure ones.

Chapter 4 considers applications to learning. In this case the problem is to supplement Axiom 1, which is static in character, with additional dynamic postulates concerning changes in choice behavior over trials. These postulates are stated in terms of transformations of the v-scale defined above. Axioms for what are called the alpha, beta and gamma models are given. The alpha model is linear in the transformations of response probabilities from trial to trial; the beta model is linear in the transformations of the v-scale but not of response probabilities. The gamma model primarily differs from the beta model in having imposed on it a boundedness condition for the v-scale. The alpha model is not new, being the linear response model much studied in the past few years. Most of the chapter is actually devoted to properties of the beta model, which originates with the author. It is premature to make an evaluation of the empirical adequacy of the beta model, but it can be reported that it is, unfortunately, difficult to work with mathematically. Moreover, it is the reviewer's own experience that in spite of its apparent attractiveness and simplicity, formulation of the model in terms of linear transformations on the v-scale is of no real help. The section on asymptotic properties of the beta model is rather incomplete, and progress since the book appeared on determining asymptotic and other properties has not essentially depended on use of the v-scale.

Chapter 5 consists of a brief summary and conclusion. The first appendix gives alternative forms of Axiom 1, the second determines the form of the latency distribution for a certain class of choice situations by using a continuous analogue of Axiom 1. The third appendix derives maximum likelihood equations for the two-alternative, two-outcome beta learning model; the results here are mainly due to Robert R. Bush. The fourth appendix lists a number of open problems, some of which are conceptual and empirical, and some of which are mathematical. I particularly liked the idea of this appendix. It is especially desirable in a rapidly developing subject to have a current summary of open problems.

I conclude with two general remarks. Luce states at the beginning that his theory of choice behavior is in a sense orthogonal to stimulus-response psychology because the basic assumptions are about relationships between choice sets and not connections between stimuli and responses. He goes on to say (p. 2), "Such an approach seems to merit careful consideration, since several decades of such S-R psychology

have not resulted in notably simple laws of behavior." My objection to this view is that Luce's own postulates do not provide a schema of the elementary processes which determine behavior. It seems much more pressing to ask at once why does his Axiom 1 hold (when it does) rather than to ask why in the nature of things stimuli become conditioned to responses. The reason it is more pressing is that *S-R* theory represents scientific conceptualization at a deeper level. One has the feeling that the schematized processes of stimulus sampling and conditioning correspond in a rough way to the dynamic physical processes of an organism in a choice situation; in contrast Luce's own theory rides on the surface of observable phenomena and postulates no underlying processes. I do not mean to suggest that Luce's own work is not important. It is just that I do not think it is really an alternative to *S-R* theory. To emphasize this point, I mention that a simple derivation can be given of Axiom 1 from the fundamental postulates of *S-R* theory as formulated by Estes and me if the additional postulate is added that the conditioning parameter associated with a possible response is independent of the particular subset of possible responses available for choice. Admittedly this postulate is also very much in the spirit of the general principle of independence of irrelevant alternatives, but unlike Axiom 1 it does not involve simultaneous consideration of several subsets at once, but is concerned primarily with *S-R* connections.

My second remark is that a more detailed statistical treatment of the empirical adequacy of the theory would have been desirable. The data that are cited too often provide suggestive rather than anything like definitive evidence in favor of the theory. The coverage of the book is qualified in the title by the phrase "theoretical analysis," but it is a happy fact that in experimental psychology it is hard to get a hearing for a theory without some kind of supporting data, a situation which too often does not obtain in several branches of the behavioral sciences. This criticism is meant to be a mild one, for I know that Luce and his associates at the University of Pennsylvania are now hard at work on experimental tests of the theory on several fronts and are also struggling with the difficult statistical problems which arise in testing many of the applications mentioned above. There is little doubt that the present book is an important contribution to the increasingly large literature on choice behavior.