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BOOK REVIEWS

AXIOMATIC SET THEORY. By PAUL BERNAYS and ABRAHAM A. FRAENKEL. Amsterdam, North-Holland Publishing Company, 1958. Pp. viii, 226. Guilders 22.50.

FOUNDATIONS OF SET THEORY. By ABRAHAM A. FRAENKEL and YEHOSHUA BAR-HILLEL. Amsterdam, North-Holland Publishing Company, 1958. Pp. x, 415. Guilders 42.

The book by Bernays and Fraenkel is divided into two parts. Part I is a thirty-eight page historical introduction by Fraenkel. It is primarily concerned with the development of Zermelo-Fraenkel set theory since the original paper by Zermelo in 1908. Part II, written by Bernays, occupies the remainder of the book. Axiomatic set theory as a formal system is presented in this part in a form similar to that given by Bernays in a series of articles published in the *Journal of Symbolic Logic* between 1937 and 1954. In the journal articles the axioms were not embodied in a fully formalized system. Two other differences in the new presentation are the occurrence of class variables only as free variables and the elimination of existential axioms by the introduction of new primitive symbols. The axiom of choice is the only axiom which is existential in form. In addition to general set theory, ordinal and cardinal number theory as well as the elements of mathematical analysis are developed from the axioms.

Both parts of this monograph are strongly recommended. The first part provides an excellent historical introduction for readers who desire a general orientation in axiomatic set theory. The second part makes available in one place Bernays' system of set theory in an improved form. I have just two minor criticisms to make. Part II badly needed the work of a good copy editor before publication; unidiomatic phrases and grammatical mistakes occur on almost every page. Secondly, the new primitive concepts introduced to eliminate existential axioms are not explained intuitively as clearly as they could be, prior to the formal statement of the axioms.

The longer book by Fraenkel and Bar-Hillel affords a leisurely and detailed introduction to the foundations of mathematics, with particular emphasis on set theory. The five chapters are concerned with the antinomies, the axiomatic foundations of set theory, the theory of types, intuitionism, and metamathematical approaches, in this order. An enormous amount of material is discussed, at least cursorily, and a very lengthy bibliography is included. This bibliography, together with the one given in Fraenkel's book *Abstract Set Theory*, provides for most working purposes a complete bibliography of the subject up to the date

of publication. To give some indication of the breadth of coverage, I shall just note the main topics in Chapter III on type-theoretical approaches: simple and ramified type theory; Quine's method of stratification; Wang's system; Lorenzen's operationist system; set theories based upon nonstandard logics (Leśniewski, Chwistek, Fitch, and so on).

Necessarily with this kind of coverage most of the topics discussed are not pursued in great technical detail. It is, however, precisely this expository emphasis that makes the book a valuable one for the philosopher. The references range so widely that even mathematicians or philosophers specializing in logic will discover remarks about papers not previously known to them.

The most technical and detailed chapter is the second one on the axiomatic foundations of set theory. The reading of this chapter would serve as an excellent introduction to Part II of the monograph by Bernays and Fraenkel. The discussion of the axiom of choice is particularly detailed and informative. It is probably the best reference in the literature for philosophers interested in finding out what the axiom of choice is all about.

In reviewing a volume of this sort it is easy enough to find omissions or inclusions to quarrel about. I shall just mention two, both minor. First, I would like to have found a more extensive treatment of the theory of definition, particularly in the context of the discussion of impredicativity. Secondly, in view of the authors' notably clear expository style, I was disappointed that they did not attempt to expound in some detail Kreisel's conception of finitistic methods. The many careful expositions of other authors are a delight to read and too numerous to mention.

PATRICK SUPPES

Stanford University