COMPUTER-BASED MATHEMATICS INSTRUCTION:

The First Year of the Project
(1 September 1963 to 31 August 1964)

by

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1. Introduction. In the past year, preliminary experiments have been conducted in preparation for the completion of the Computer-based Laboratory for Learning and Teaching at Stanford. This report describes the equipment available in the laboratory, outlines the work which has been done on the programs for first grade, fourth grade, and sixth grade (mathematical logic), and describes the program logic to be used on the computer.

Two groups of six first-grade children will be at Stanford five days a week for twenty minute sessions beginning in October, 1964, although the laboratory will not be fully functioning until February or March 1965. A few selected fourth-grade students will also be run throughout the year. Both the first - and fourth-grade materials follow the general content and sequence of the Sets and Numbers books (1A and 1B, 4A and 4B). The fourth-grade material, however, is constructed for use by children who have had no previous introduction to any of the new mathematics programs.

The mathematical logic program, which has been run in a slightly different form during the summer of 1964, will be presented to a group of gifted sixth graders beginning in November.

2. Description of the laboratory. The system consists of a medium-sized computer and six subject stations. Each station is located in a small (7ft. by 8ft.) room and contains terminal devices which permit communication between the computer and the user. The user may be one or more of the following: programmer, operator, experimenter, student, or experimental subject. The computer controls the presentation of both visual and auditory material; the subject responds with a light pen, a typewriter keyboard, or a microphone.

2.1 Optical display unit. The optical display unit is a rapid, computer-controlled, random access projection device which presents visual material to the user on a 10 in. by 13 in. ground glass screen. The source material (any 8½in. by 11 in. page of text material) is photographed on microfilm and is stored in small projector cells with a capacity of 256 pages each.

Each display unit has two projectors which project onto a single screen giving a total capacity of 512 individual pages.

However, many more combinations are possible by constructing composite images from both projectors on the common screen. Each projector has its own shutter that can be opened or closed and the screen is divided into eight equal sections each with its own individually controlled mask.

The user responds to the display and sends information to the computer with the light pen. As the pen is touched to the screen, the coordinates of that position are sent to the computer and may be compared with any prearranged areas of the screen. The accuracy of the light pen will permit the identification of any ¼ in. square on the screen. Most of the operations of the display unit will occur in approximately one second. A light in the pen is turned on to indicate acceptance of the information by the computer.

2.2. Cathode-tube display. This display unit is a cathode ray tube, commonly called a "scope", which can display points of light in an area 10 in. high by 10 in. wide. There are 1024 possible positions on both the horizontal and vertical axes. In addition to individual points there are 120 prearranged characters which may be displayed in five different sizes. It is also possible to display vectors by simply identifying the end points. An electronic light pen is available for sending information from the user back to the computer by identifying points on the face of the tube.

2.3 Typewriter keyboard. A typewriter keyboard is attached to each scope and may be used to send information from the student to the computer.

2.4 Audio system. The audio system has the provision for playing pre-recorded messages to the user. The messages are recorded on 6 in. wide magnetic tape and two tape transports may be assigned to each of the six user stations. Each transport has a capacity of about 17 minutes which can be divided into any combination of messages from one message 17 minutes in length to 1020 messages of one second each. The user may also record onto a selected portion of the tape and then have it played back for comparison purposes or retained as response data. The various control and switching functions between the tape transports, microphones, and speakers are performed by a small process control computer. The random access time to any stored message is approximately one second. The audio system will not be installed until February, 1965.

2.5 Central computer. The main control functions of the entire system are handled by a medium-sized computer. The main memories are a 16,000 word core, and a 4,000 word core which can be interchanged with any of 32 bands of a magnetic drum. All input

output devices are processed through a sequence break system which services them only when necessary. Two high-speed data channels permit simultaneous computation and servicing of peripheral devices. Additional backup in computational power, additional storage, and increased input-output speed is obtained through connections to a larger computer system with magnetic tape and disk storage, located at the Stanford Computation Center.

3.1 First-grade program: preliminary experiments. Before beginning the preparation of material for use in the computer-based laboratory, an experiment with twenty-nine kindergarten children was conducted. These children comprised an entire kindergarten class with the exception of one mentally retarded boy. The children work individually, with one experimenter and one child present at each session.

The experiment used the material that we plan to use in presenting the first six concepts on the first-grade program, namely a section on:

1) learning to use the light pen, including in the directions words needed in later sections:

box	first	left
top	last	right
middle	arrow	dotted line
bottom	goes with	

2) using concrete objects to show sets
3) using set notation to picture a set
4) the empty set
5) equal sets (choosing an answer from two choices)
6) equal sets (more objects in the sets, three answer choices)

The material for the experiment was arranged in the form of several 8½ in. by 11 in. booklets requiring one response per page. The children responded by putting a dot with a crayon on the place indicated by the directions. (This corresponds to the use of a light pen to point to the correct answer on the optical display unit.)

A tape played to each child gave directions for the required responses. For all but the second section, each audio message was from five to ten seconds in length and called for a response from the child at the end of the message. In the second section, there were some longer messages accompanying displays of concrete objects.

The experimenter ran the tape recorder, located the page on which the child was to respond, and noted any comments the child made or any questions he asked. The experimenter did not make any remarks pertaining to the material unless the child did not understand an instruction. Then, the experimenter repeated the instruction, reading from the script used to make the tape. If the child

still did not understand the instruction, the experimenter made an explanation, noting exactly what was said.

The material was constructed so that each concept was presented and followed by a group of problems. If a certain number of responses to these problems was incorrect, the child went to a "branch". In the branch, the concept was presented again in a slightly different way and a new set of responses was made by the child.

If the child made enough correct responses on the first presentation of a concept, the branch was skipped and the child went on to the next concept. At the end of some sections (including concept and branch) there was a short review section.

Some results of this experiment were as follows:

1) Of the words introduced in section 1, "last", "left", and "right" were the most difficult. There was no branch in this section.
2) When there was an explanation given using concrete objects, the children became very restless and their attention strayed if the audio message was over ten to twelve seconds long without asking them to make a response. There was no branch in this section.
3) Using set notation to picture sets was quite simple. Eight of the twenty-nine children had enough errors to go to the branch.
4) The concept of the empty set had the fewest number of errors. Four children made enough errors to go to the branch.
5) The concept of equal sets proved to be the most difficult. Nineteen children made enough errors to go to the branch. The branch for this concept was written in a slightly different way, and proved much simpler. The wording of the directions for responding to the problems seemed to cause most of the difficulty. With the different wording of the branch, the number of errors substantially decreased. However, both the concept and branch were much more difficult than they should be, so these sections have been substantially rewritten for the optical display units.
6) The section on equal sets, using a somewhat different format and wording from either the concept or branch for the previous section was substantially simpler, with five children having to go to a branch. In the review for sections 5 and 6, fifteen of the children made no errors.

3.2 Instructional material. Since the user may make a response only by touching the light pen to the screen, there are certain limitations on the kinds of formats that can be used. And although

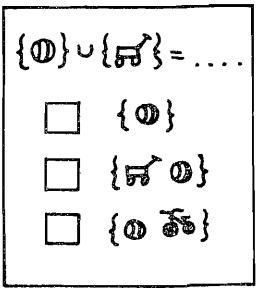
some material in the Sets and Numbers books has had to be rewritten in order to fit a multiple-choice or simple keyboard format, almost every concept in the books has been prepared for the optical display units.

The material follows the outline in the books and is essentially:

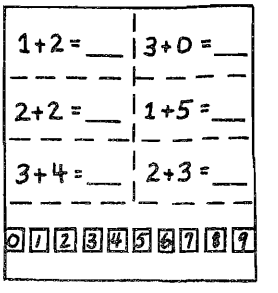
introduction to display unit and use of light pen
set notation
empty sets
equal sets
union of disjoint sets
properties of sets
N notation and number of things in a set
arabic numerals
counting from 1 to 9
addition (equation form, column form, sequential problems, variables)
difference of sets
10 to 100
place value
first to sixth
1-100 by ones, by tens, by twos, by fives, by threes, by fours
number words, zero-nine
1/2
subtraction, equation form, column form
two variables
time, money, calendar, cups, pints, quarts, dozen, ½ dozen in., ft.
geometry (not yet completed)

Because there is to be a minimum of audio message and separate classroom instruction, the number of problems and display pages has been increased. Some of the basic formats are as follows:

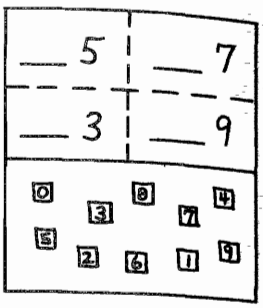
1) The child is asked to perform the union of two sets and to show the correct answer by placing his light pen in the box next to that set.



2) For each problem, the child adds the two numbers, and puts his light pen on the correct answer on the keyboard below.

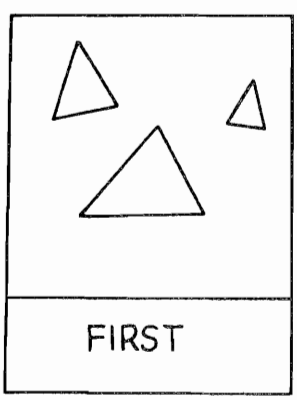


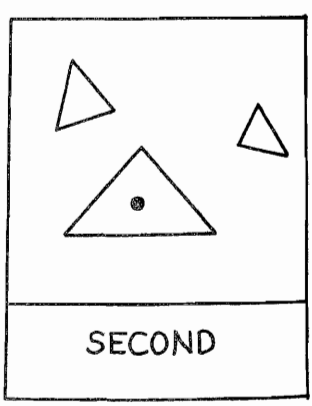
3) The child sees the first problem and is told to put his light pen on the number which "comes before". The format of the keyboard has been changed for this type of problem.



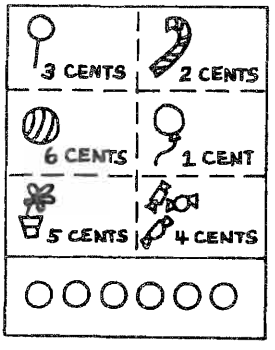
In the first example, the child is simply given the next problem after a correct response. If he responds incorrectly, an X is projected across the box to which he pointed and is then removed, and the child is given another trial. For each of the formats 2) and 3), the child sees only one problem at a time and the keyboard at the bottom of the page. The correct answer will be given by the second projector after the child has 1) made a correct response so that he can see the answer he gave on the screen or 2) made a pre-determined number of incorrect responses and is to go on to the next problem.

4) This example was taken from the presentation of "first, second, third" and the concept of ordering. The child is instructed to order the triangles from largest to smallest. The word "first" is shown at the bottom of the page from the second projector, and the child touches the largest triangle with his light pen. A correct response causes a dot to appear on the triangle, and then the word "second" replaces "first" at the bottom of the page. He then repeats the process.





5) The concept of counting money is shown in this example. The first problem is shown to the child and he is told to put his light pen on the correct number of pennies starting at the left. A correct response sends him to the next problem. An incorrect answer causes the shutter to be closed for a moment and then the child is given a second trial.



Some of the material dealing with numbers is better suited to the scope since the user can type answers and the material he types can appear in the problem shown on the scope. For this reason, we plan to prepare the material for some concepts for use on the scope in addition to preparing the same material for the optical display units. Eventually some children will use the optical display units while others will use the scope. The results will be compared to determine whether the type of response (multiple choice or constructed response) affects the child's learning.

3.3 Program logic. The first-grade program is organized very simply at this point, and makes use only of the optical display unit. It consists of approximately fifty sections, each introducing a new "concept", followed by problems to be worked. The number of problems varies considerably from concept to concept. A criterion of successive-number-correct has been set for each section. If the student meets the criterion he proceeds immediately to the next concept (or to a review section if scheduled); if he cannot meet the criterion he finishes the section and goes into a remedial branch which presents the same concept in a new way. Eventually a second remedial branch will be included for the student who still has not acquired the concept. For the first run of the material, however, a teletype printout will indicate to the experimenter that a child is making continuous errors and the experimenter will then instruct the child personally. (Criteria to be included later in the program are total number of errors, latency, and weighting of different problems.

These data are now recorded by the computer for use in analyzing the progress of the students.) A short review section has been written for each concept. As the child proceeds through the material, a group of four review problems from each of four or five previous concepts will be presented to him. If he responds correctly to two of the four problems he will continue through the review to the next concept. If he makes more than two errors, the computer will register these data, and will present an extensive review of the particular concept to the child on the next day. In this way, the child will be tested continually on his retention of previous concepts.

4.1 Fourth-grade instructional material. The program is being written for the "average" student who has had no previous contact with "modern mathematics" or with the concept of a set. The first unit of the program begins with a review of:

1. place value
2. addition
3. subtraction
4. multiplication
5. division
6. word problems and equations

The second unit develops the following topics:

1. operations on numbers
2. commutative law
3. associative law
4. stating reasons for deductive steps in problem solving
5. distributive law

Of the two units started, the first is complete and ready to be programmed. The second is approximately 20% complete as of this date.

4.2 Program logic. Each instructional unit is broken into a number of learning blocks which consist of the presentation of a major concept and problems to be worked by the student. The format of the program is illustrated on the following pages. The first chart shows the different ways in which students of varying abilities will proceed through the material and gives an indication of the criteria for moving ahead or back to remedial material in the program. Each "main line" concept will consist of:

1. an introduction
2. explanations of concept
3. one or more sample problems
4. further exercises ranging from simple to more difficult

If a student makes an error while working a problem, he is given a clue so that he may correct the error before continuing. If all

clues are exhausted and the student has still answered incorrectly, then he is presented with three or four remedial problems of the same type. After the remedial problems are solved or the correct solution has been presented, he returns to the original problem. Advanced problems have been constructed for the bright child. If this student meets a certain criterion in the main-line block, he skips to the advanced problems. As shown on the chart, it is possible for a student to progress to these more difficult problems without ever having to return to the "average" blocks; he will receive the introduction, explanation and sample problems, and then (provided he has met the criteria of previous lessons) go directly into a block of advanced problems. The second chart presents a more detailed description of the material in one concept block and its remedial and advanced branches. The twelve problems of one concept are displayed. A set of clues is given for the first problem, and typical remedial problems are shown at the far left. Two sets of advanced problems are also shown to demonstrate the increasing difficulty of these exercises. A special orientation unit is being prepared to familiarize the student with the response capacities of the scope and the optical display unit so that when he is presented with a problem (e.g.: add: 34 + 47 = ___) he will respond in a prescribed manner. (In this example, 1C18. This represents the following train of thought: 4 + 7 = 11; record the 1, carry the 1 ten; 3 tens + 4 tens + 1 ten = 8 tens; thus the answer is 81.) We have had the students type in "C" for carry and the number he wishes to carry to the next column. An alternative way would be to restructure the problem by typing in 34 + 47, 1C18, and the problem would appear on the scope in the following format:

```
  1
 34
+47
 81
```

As seen in this second mode of response, 10 characters were required to demonstrate a solution. A simple subtraction problem requires careful consideration on the part of the student as he responds:

```
subtract:  1317
           247
         - 48
           199.
```

Response: R47, 3, 17, 9. R23, 1, 13, 9, 1. The thought process behind this response is: Regroup 47 into 3 tens and 17 ones, 17 - 8 = 9. Regroup 23 into 1 hundred and 13 tens, 13 - 4 = 9. 1 - 0 = 1. This type of response corresponds directly to the sequence of steps taken by the student to solve a particular problem. Thus, once the typewriter "code" is acquired by the student, the

Chart 1

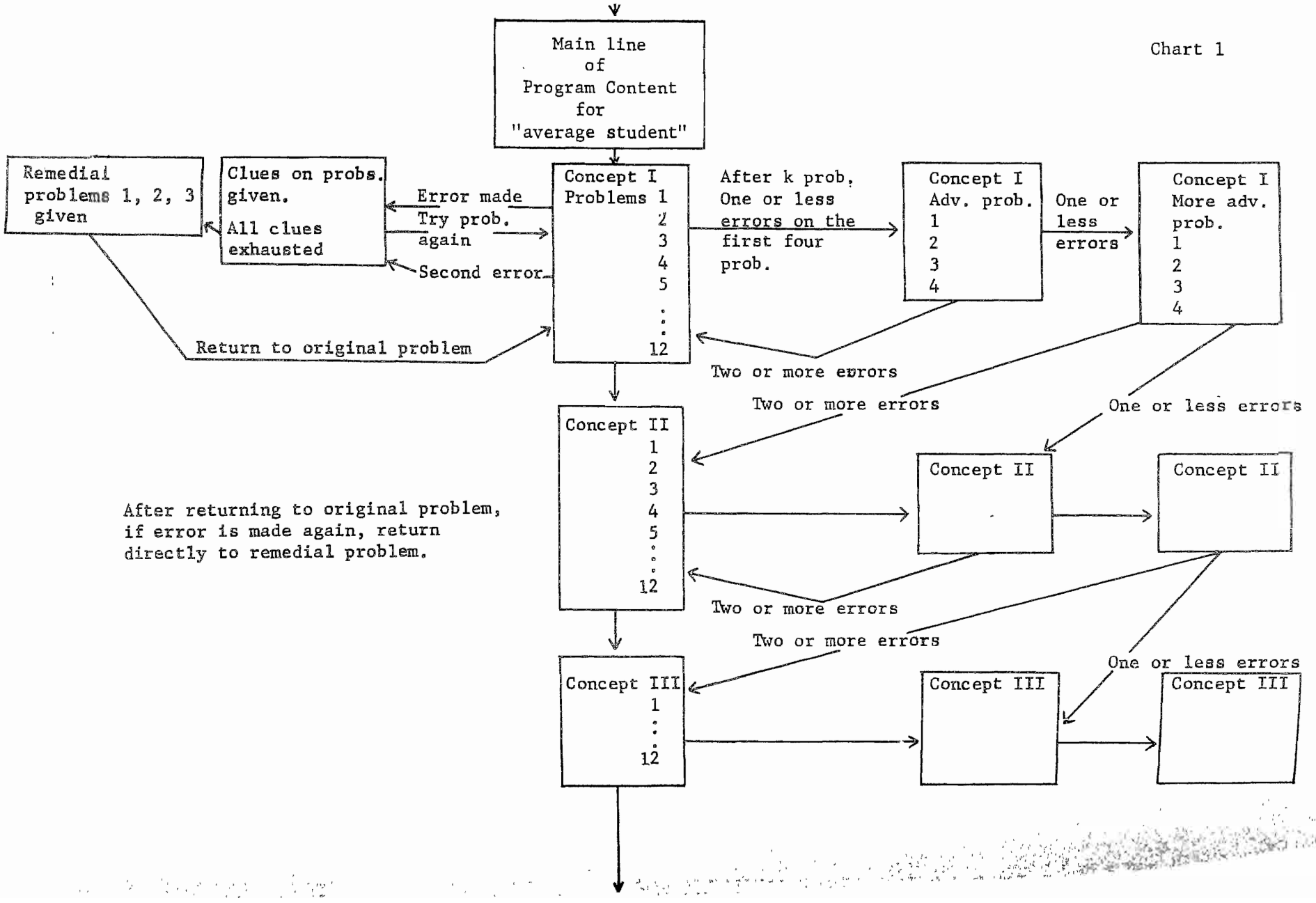

Main line of Program Content for "average student"
Concept I Problems 1 2 3 4 5 . . . 12
Error made
Try prob. again
Second error
Clues on probs. given.
All clues exhausted
Remedial problems 1, 2, 3 given
Return to original problem
After k prob, One or less errors on the first four prob.
Concept I Adv. prob. 1 2 3 4
One or less errors
Concept I More adv. prob. 1 2 3 4
Two or more errors
Two or more errors
One or less errors
Concept II 1 2 3 4 5 . . . 12
Concept II
Concept II
Two or more errors
Two or more errors
One or less errors
Concept III 1 . . . 12
Concept III
Concept III
After returning to original problem, if error is made again, return directly to remedial problem.

Chart 2: Sample Concept in Main Line

The Distributive Law for Multiplication

REMEDIAL PROBLEMS (4)

4 + 3 = ___

5 x 4 = ___

5 x 3 = ___

5 x 7 = ___

CLUES

1. Check your operation symbols
2. Check your grouping symbols
3. Check your addition and multiplication facts

Sample of Choices

1. (5 + 4) x (5 + 3)
2. (4 x 5) + (4 x 3)
3. 20 + 7
4. (5 x 4) + (5 x 3)

Criteria

k = 4
m = 3
n = 4

1. m = 5 x (4 + 3) (multiple choice on optical display unit and constructed on scope)
2. n = 3 x (5 x 4)
3. p = (2 + 6) x 4
4. q = 6 x (3 + 2)
5. r = 8 x (4 + 5)
6. s = (5 + 7) x 7
7. t = 9 x (6 + 8)
8. u = (7 + 5) x 8
9. v = 5 x (6 + 5)
10. w = (9 + 6) x 4
11. k = 3 x (8 + 7)
12. j = (6 + 7) x 6

ADV - I

1. a = 12 x (7 + 8)
2. b = 12 x (10 + 11)
3. c = (9 + 8) x 12
4. d = 12 x (10 + 7)

ADV - II

1. 9 x (30 + 6)
2. 12 x 24
3. (40 + 7) x 12
4. 25 x 45

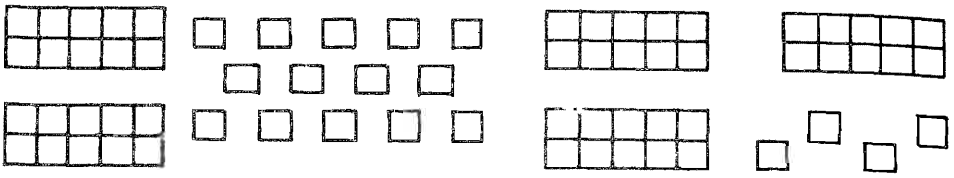
machine data can indicate exactly where he has made his mistakes.

The following examples taken from the lesson material reflect the versatility of the scope:

1. 2321 is _____thousands _____hundreds _____tens ___ones

An arrow appears over the blank to show the desired order for responses

2. squares move to this form



__2__ tens and __14__ ones = __3__ tens and __4__ ones.

3. Mary and Tom together have the same amount of money as Carol and Bill together.

Mathematical sentence: m + t = c + b (typed in by student)

5. Mathematical logic program. Experiments in past years show that gifted elementary school students taught by teachers trained in logic can score better than 90% as high on logic tests as Stanford University logic students. In view of these test results, a program for instruction in mathematical logic has been prepared for gifted sixth graders which makes full use of the versatility of the computer-based laboratory. The scope in this laboratory has the capacity for immediate evaluation of response data and for the acceptance of any valid step in a proof, making possible the presentation of new stimulus items which are contingent upon the student's immediately previous performance.

5.1 Preliminary experiments. In the spring of 1964 some of the most frequently used rules of sentential derivation were programmed for presentation on the scope. Several sixth-grade students who were soon to complete two years of logic study were given written instructions and 21 derivations to be carried out. They took readily to performing the derivations at the keyboard even though there was some change from the symbolism they had been using. It was clear that the mechanics of operation was no handicap.

A second experiment was conducted with two boys who had just completed fifth grade. One, a gifted child, had had classroom instruction in logic. The second boy was above average intelligence and had not been instructed in logic. During the summer

these students completed 21 lessons; the first in 32 sessions averaging 29 minutes each, the second in 38 sessions averaging 37 minutes each. Continuous observation was made and a record kept of questions asked by the students and of additional instructions and hints offered by the instructor. The lessons covered almost precisely the material that had been covered before by the experienced student. Nevertheless, he did not seem to find the work boring. Thus we could presume that sufficient drill to achieve a desired degree of overlearning does not offer a serious motivational problem. By the completion of lesson 21 the inexperienced student was doing work comparable to that of the experienced student.

5.2 Instructional material. At first, the student is presented with logical symbols which are not interpreted for him. He is to view the symbols as concrete objects. Using arbitrary rules of derivation introduced one at a time, the student manipulates the symbols as he would pieces on a game board, in order to arrive at a required conclusion. Thus, the operations are made on a concrete level and the symbols or "pieces" are not looked upon as being abstract carriers of "meaning" or as representing something other than themselves.

At the next stage, the "pieces" are regarded as carriers of the possible truth values of True or False. Given a truth value for each atomic sentence, the student constructs diagrams to determine the truth value of complete molecular formulas. (Atomic formulas are the simplest type of formula, and have a single, important property - their truth values. They combine with symbols called connectives to form more complex formulas (molecular formulas). Connectives make true or false claims about the truth values of the atomic formulas. In this way, the student understands that the truth value of a molecular formula is a function of the truth values of the included atomic formulas, and, consequently, that the connectives are "truth-functional".) The student is shown that the rules initially introduced as arbitrary, never allow us to derive a false conclusion from true premises.

As the next step, the student constructs truth tables which give the truth value of a molecular formula under every possible combination of truth values for the atomic formulas of which it is comprised. Now it is possible to demonstrate the validity of each rule. The student learns to define a tautology as a molecular formula that is true under all possible combinations of truth values for its atomic formulas. The truth-functional definitions of the connectives in the single formula corresponding to an arguement are analyzed to show that an argument is valid if and only if it is a tautology. From this point on, the validity of all new rules is demonstrated when the rule is introduced. These include the methods of conditional proof, indirect proof, and biconditional replacement.

Finally the formal concepts of invalidity of arguments, and consistency or inconsistency of sets of premises, are studied and methods for their proof developed. Some metalogical discussion is given on the development of a logical system and how the computer applies it.

A spiral approach is followed in the lessons: only five rules of derivation are introduced before presentation of the concept of truth in lesson eight. In that lesson the concept of validity is introduced and several rules are immediately justified. Then truth-functional definitions are presented in truth tables and are used immediately to develop English approximations for the connectives. English examples are used as illustrations. Full translation of English molecular sentences is postponed until lesson twenty-seven.

5.3 Program logic. The program is logically broken into two parts: 1) the construction of proofs and 2) a general scope program which is used for presenting additional material.

During the construction of proofs the scope is used as a worksheet on which the steps of the derivation are carried out. The following example shows a problem as it is seen on the screen.

Derive R:

P	(1)	$P \rightarrow Q$
P	(2)	P
P	(3)	$Q \rightarrow R$

At this point the student responds by typing "AA 12 TAB KEY". Each character is displayed on the scope as it is typed. Errors may be corrected by pressing the backspace key which causes the last character entered by the student to be "erased".

The student's input is them edited for errors in "grammar" and he is required to correct any errors found. When the form of the line is correct the validity of the construction requested is checked. The program will then respond with the corresponding formula or give the student an explanation for his mistake. No attempt is made to determine the strategic appropriateness of the steps. Thus in problems with any degree of complexity there are many ways of arriving at the desired conclusion and the path taken is left completely to the ingenuity of the student.

As the program has progressed through stages of pilot runs, most of the rules have been expanded and have become more abstract in their definitions and response characteristics. Many of the rules allow for the introduction of formulas by the student, i.e.

premise introduction for conditional and indirect proofs. The example given above would appear as below when completed. Student input is underlined in the example:

Derive R:

P	(1)	P Q
P	(2)	P
P	(3)	Q R
AA 1 2	(4)	Q
DA 3 4	(5)	R

CORRECT

The general scope program consists of the following:

a) English text display with or without missing words to be typed in by the student.

b) Multiple choice questions. Responses are made by pointing to a displayed box with the light pen. Incorrect responses cause the box pointed to be replaced by an X and the student then proceeds to select another answer.

c) Filling in the truth assignments for a formula truth table. A small T and F (TRUE, FALSE) are displayed with each desired entry. The student then responds by pointing to one or the other. If a correct response is made the other choice is dropped. If an incorrect response is made the program flashes a "NO!" for approximately two to three seconds at the point of error and then replaces the entry with the correct response (since it is the only choice left) in high intensity. When the exercise is completed all incorrect responses are displayed with a greater intensity to draw the student's attention to those entries.

Construction of truth tables is introduced by having the student assign all possible truth assignments to a formula repeated several times on the scope and then the program slowly merges the formulas and truth assignments to form the familiar truth table.

d) An exercise in removing unnecessary parentheses is done by displaying a formula which contains parentheses that could be removed without changing the meaning of the formula. This is done by displayed "pointers" which follow the light pen. Thus the student points to the parentheses which he thinks can be removed. The program checks his request for validity and responds with appropriate error messages or the parentheses fade from the scope and the formula slowly closes the gaps left. This process is continued until the student indicates he believes no more may be removed. The program then proceeds to the next problem or tells the student that there are still parentheses left which he must remove.

e) Exercises in pointing out the order of dominance of connectives in formulas are done by displaying boxes above each connective and having the student point to these in the required order. Errors cause the box indicated to be replaced by an X; correct responses cause the box to be replaced by the appropriate numeral indicating the order of dominance.

6. Project staff. The project director is Dr. Patrick Suppes, Professor of Philosophy and Statistics, Stanford University. Approximate staff allocations to the various programs are as follows. The first-grade program: Mrs. Dolly Kyser, Miss Anne Hyslop; fourth-grade program: Mr. Max Jerman, Miss Phyllis Cole; mathematical logic program: Mr. Frederick Binford, Professor Dana Scott; chief computer programmer: Mr. Dow Brian.

7. Project support. The computer-based laboratory for learning and teaching has been constructed primarily from a grant from the Carnegie Corporation of New York. The computer-based mathematics instruction (CBMI) project is supported by a grant from the Course Content Improvement Section, National Science Foundation.